A Computer Program to Analyze Optical Parametric Up-Conversion Processes in Nonlinear Crystals

R. A. ANDREWS

Quantum Optics Branch
Optical Sciences Division

October 27, 1969

NAVAL RESEARCH LABORATORY
Washington, D.C.

ABSTRACT

A Fortran computer program has been developed which analyzes a nonlinear material to determine (a) the range of IR wavelengths that can be converted in a phase matched (PM) process, (b) the PM orientation of the wave vectors for critical and noncritical PM, (c) the angular aperture for PM conversion, and (d) the maximum number of resolvable lines for image conversion. These characteristics are determined as a function of IR wavelength for a given pump wavelength, pump radiation divergence, and length of nonlinear crystal.

PROBLEM STATUS

This is an interim report; work on the problem is continuing.

AUTHORIZATION

NRL Problem N01-14
Project XF-52545002-8063

Manuscript submitted May 27, 1969.

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INTRODUCTION

Infrared (IR) light can be up-converted to visible light, via a parametric process, in crystals which have a nonlinear susceptibility (1-3). In the second-order processes considered here, IR light of frequency ω_{IR} is mixed with intense light of frequency ω_P, called the pump light. The result is signal light of frequency $\omega_S = \omega_{IR} + \omega_P$. For the process to be efficient, the crystal must be transparent at the three frequencies involved, and momentum must be conserved; i.e., the process must be "phase matched" (PM). A process is PM if the directions of propagation in the nonlinear crystal for the three frequencies of light can be found such that the signal wave vector equals the sum of the pump and IR wave vectors, i.e., $(\mathbf{k}_P + \mathbf{k}_{IR}) - \mathbf{k}_S = 0$.

Given a nonlinear crystal, it is necessary to find the range of IR wavelengths that can be up-converted in the material and to find the characteristics of the up-conversion process as a function of the IR wavelength. Up-conversion of IR to the visible necessitates the use of an intense source of laser light as a pump whose wavelength is near the long-wavelength edge of the visible spectrum. Once the wavelength of the pump source has been specified, the range of ω_{IR} for PM up-conversion can be determined. Then for each set of frequencies ω_P, ω_{IR}, and ω_S, the orientation of the crystal with respect to the wave vectors that gives a PM up-conversion, i.e., the phase match angles, can be found. In each case the wave vectors can be either collinear or noncollinear, and both situations must be considered. Also, if the IR to be up-converted carries spatial information such as an image, one must determine such things as the acceptable angular aperture for PM up-conversion, the resolution limit* for the beam divergence of a given pump, and the orientation which maximizes the angular aperture. Further, all of the characteristics of a PM up-conversion process must be calculated as a function of ω_{IR}, the IR frequency.

Finding the PM angles and other characteristics of an up-conversion process means solving transcendental equations. To obtain any desired accuracy the complete analysis is best done on a computer using an iteration technique. This report describes a Fortran IV program which does that analysis. The characteristics of the nonlinear crystal necessary for the analysis are its limits of transparency and its refractive indexes as a function of wavelength.

The next section describes the mathematical basis of the calculation. This is followed by a description of the program itself and two appendixes which give a listing of the program, sample input data, and the corresponding output.

MATHEMATICAL FORMULATION

The requirements for PM up-conversion (4) are

$$\omega_P + \omega_{IR} = \omega_S \tag{1}$$

and

*The resolution limit considered here is the particular case of an extreme multimode pump source with the IR image and pump source optically at infinity with respect to the nonlinear crystal. See, for example, "IR Image Optical Parametric Up-Conversion," R.A. Andrews, IEEE J. Quant. Electr., Jan. 1970.

$$\mathbf{k}_P + \mathbf{k}_{IR} = \mathbf{k}_S , \quad (2)$$

where the subscripts P, IR, and S refer to pump, infrared, and signal, respectively, and $\mathbf{k}$ is the corresponding wave vector. For a crystal of length L, PM up-conversion takes place as long as

$$\Delta k = |\Delta \mathbf{k}| = |\mathbf{k}_P + \mathbf{k}_{IR} - \mathbf{k}_S| \le \frac{2\pi}{L} , \quad (3)$$

where for convenience we assume that $\Delta \mathbf{k}$ is measured along the direction of $\mathbf{k}_S$ as shown in Fig. 1. In general, the orientation of the nonlinear crystal for PM up-conversion is given by the solution to the following equation:

$$\Delta k = \left[\frac{n^i(\omega_P, \theta_P)}{\lambda_P} + \frac{n^j(\omega_{IR}, \theta_{IR})}{\lambda_{IR}} \cos\phi \right] \sec\rho - \frac{n^k(\omega_S, \theta_S)}{\lambda_S} = 0 , \quad (4)$$

where

ϕ = angle between $\mathbf{k}_P$ and $\mathbf{k}_{IR}$

ρ = angle between $\mathbf{k}_P$ and $\mathbf{k}_S$

θ_ℓ = angle between the Z-axis and $\mathbf{k}_\ell (\ell = P, IR, S)$

$i, j, k = O$ or E for extraordinary or ordinary polarization

It is assumed that the parametric process is confined to the yz plane, where x, y, and z are the optical axes of the nonlinear crystal, and z is the optical axis for a uniaxial crystal. However, the crystal may be optically biaxial; therefore

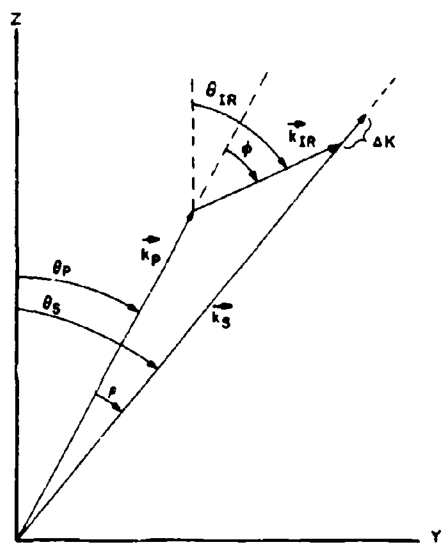

Fig. 1 - Crystal geometry for up-conversion from infrared to visible wavelengths

$$n^O(\omega,\theta) = n_x(\omega) \tag{5}$$

and

$$n^E(\omega,\theta) = \left\{\frac{\sin^2\theta}{[n_y(\omega)]^2} + \frac{\cos^2\theta}{[n_z(\omega)]^2}\right\}^{-1/2}, \tag{6}$$

where n_x, n_y, and n_z are the refractive indexes for light polarized along the x, y, and z directions.

Solutions to Eq. (4) and, in particular, collinear up-conversion ($\phi = \rho = 0$) represent examples of critical phase matching. The noncritical phase matching (NCPM) solution (Ref. [) requires that

$$[\partial\Delta k(\theta_P,\phi)/\partial\phi]_{\theta_P} = 0 \tag{7}$$

The crystal has finite length L and up-conversion will take place whenever $\Delta k < 2\pi/L$. Hence, for a fixed pump beam direction, there will be a range of values of ϕ for which IR light will be up-converted. This range defines the angular aperture α for $\mathbf{k}_{IR}$ and is defined by all values of ϕ such that for a fixed value of θ_P,

$$\Delta k(\phi,\theta_P) < 2\pi/L \tag{8}$$

or

$$\alpha = |\phi_1 - \phi_2| , \tag{9}$$

where

$$|\Delta k(\phi_1,\theta_P)| = 2\pi/L , \quad \phi_1 > \phi_0$$

$$|\Delta k(\phi_2',\theta_P)| = 2\pi/L , \quad \phi_2 < \phi_0$$

$$\Delta k(\phi_0,\theta_P) = 0 .$$

α_{INT} is the angular aperture inside the crystal. Outside the crystal,

$$\alpha_{EX} = \left|\sin^{-1}[n^j(\omega_{IR},\theta_P+\phi_1)\sin\phi_1] - \sin^{-1}[n^j(\omega_{IR},\theta_P+\phi_2)\sin\phi_2]\right| \tag{10}$$

In this manner, each PM process can be characterized by the width of the $\Delta k(\phi)$ vs ϕ curve with θ_P fixed. Hence, the half-widths are defined as

$$HW_1 = \phi_0 - \phi_1 \tag{11}$$

and

$$HW_2 = \phi_0 - \phi_2 . \tag{12}$$

The pump radiation always has a finite divergence, and hence the PM angles are not well defined. Variations in θ_P will give PM up-conversion for different values of ϕ; hence the angular aperture is increased. The divergence Δ of the pump radiation inside the crystal is, however, less than that outside, i.e.,

$$\Delta_{INT} = \Delta_{EX}/\bar{n}(\omega_P) \; . \tag{13}$$

where $\bar{n}$ is some average index of refraction.

The maximum angular aperture is obtained with NCPM. The aperture is the greatest if, in this case, Eqs. (4) and (7) are solved with $\Delta k = -2\pi/L$. This technique is discussed by Warner. However, a larger angular aperture does not always guarantee better resolution. To determine resolution, one must find the change in ϕ that will cause a variation in θ_S just equal to the width of θ_S values caused by the divergence of the pump radiation. A variation in θ_P due to a divergence of Δ causes a change in θ_S of

$$y = \Delta + \sin^{-1}\left[\frac{k_{IR}}{k_S} \sin(\phi - \Delta)\right] - \sin^{-1}\left[\frac{k_{IR}}{k_S} \sin \phi\right] . \tag{14}$$

This change in θ_S is equivalently produced by a change in ϕ (with θ_P fixed) of

$$\epsilon = \sin^{-1}\left\{\frac{k_S}{k_{IR}} \sin\left[\Delta + \sin^{-1}\left(-\frac{k_{IR}}{k_S} \sin \Delta\right)\right]\right\} \tag{15}$$

Hence, the number of resolvable lines of IR is equal to

$$R = \alpha_{INT}/\epsilon \tag{16}$$

GENERAL DESCRIPTION OF THE PROGRAM

The program is written in Fortran IV language and, in the form given in Appendix A, has been run on a CDC 3800 computer. Input to the program is minimal and is on punched cards.

For each material to be analyzed, it is first necessary to fit dispersion data to a Sellmeier equation of the form

$$n_i^2 = A_i + B_i/(C_i - \lambda^2) - D_i\lambda^2 , \quad (i = x, y, z) . \tag{17}$$

(The program can easily be modified to use any other dispersion relation by modifying the function FIN (W, I), which is given in Table 1.) Besides the constants A_i, B_i, C_i, and D_i, it is necessary to supply the upper and lower wavelength limits of the transparent region of the crystal, the length of the crystal, and the range of values of the pump-source divergence to be considered. This information is given on a set of four data cards for each material. The required format is given in Table 2. The first data card gives the number of materials to be analyzed. Data cards 2 through 5 are repeated for each material. The symbols used for the data are defined in Table 3.

Pump wavelengths of 0.6943 μ and 1.06 μ have been selected, since they lie in or near the red part of the spectrum as discussed above, and intense laser sources are readily available at these wavelengths. The program analyzes up-conversion processes in which all three wavelengths lie in the region of transparency. The IR wavelength is scanned from the lower wavelength limit of transparency upward in equal increments on a logarithmic scale. The interval is determined in statement 10 of the main program.

Table 1
Subprograms

Name	Calculations	Line No. of Accuracy Test
FNX (XX, PP, DD)	Angle between k_p and z-axis for PM where XX = angle between k_p and k_{IR} PP = (see text) DD = value of Δk at PM (see text)	222
TNCPM (TCL, D)	Noncritical PM angle TCL = collinear PM angle D = value of Δk at PM	144
PVAR (T, P, DD, E)	Variation in ϕ for PM when θ_p is varied T, P = PM angles θ_p, ϕ DD = amount θ_p is increased E = value of Δk	178
FDK (T, P)	Δk T, P = PM angles θ_p, ϕ	—
HWDK (L, T, P, I)	Width of $\Delta k(\phi)$ vs ϕ curve L = length of crystal T, P = PM angles I = index to specify type of variation in ϕ to be performed	281
OUT	Printout routine	—
FIN (W, I)	Refractive index W = wavelength of desired index I = 1, 2, or 3 corresponding to x, y, or z optical axis	—
FN1 (A) FN2 (A) FN3 (A)	Refractive index for arbitrary direction of propagation A = angle between direction of propagation and z axis	—
FN2C (Q) FN3C (Q)	Refractive index squared for arbitrary direction of propagation Q = same as A in FN1 (A)	—

Table 2
Input Data Cards

Data Card Number	Format	Symbols
1	(I2)	N
2	(A5, 5X, 6F 10.1)	ANAME, UL, LL, L, DL, DU, DD
3	(4F 10.4)	AX, BX, CX, DX
4	(4F 10.4)	AY, BY, CY, DY
5	(4F 10.4)	AZ, BZ, CZ, DZ

Table 3
Definitions of Symbols

Symbol	Definition	Units
N	Number of materials to be analyzed	Integer
ANAME	Name of material	
UL, LL	Upper and lower wavelength limits for transparency	Angstrom
L	Length of crystal	Centimeter
AX, BX, ... DZ	Parameters for Sellmeier equation for indexes corresponding to x, y, and z optical axes. (For uniaxial crystal $z \rightarrow$ extraordinary polarization, $x = y \rightarrow$ ordinary polarization.)	

For each set of possible wavelengths, all polarization combinations are checked for a possible PM process. The output is labeled "$O + O = E$," etc. for (ordinary polarization) + (ordinary polarization) = (extraordinary polarization), etc. "Not Phasematchable" is printed if an orientation cannot be found for phase-matched up-conversion. If up-conversion at a given set of wavelengths is phase matchable, then all the parameters discussed in the previous section are calculated and printed (see Appendix B).

The functions of the various subprograms are listed in Table 1. In particular, the function FNX (X, P, D) is more general than need be for this program. FNX will calculate a PM angle for any orientation of $\mathbf{k}_{IR}$ with respect to $\mathbf{k}_P$. In this case P is the angle between the ($\mathbf{k}_P$, z-axis) plane and the ($\mathbf{k}_{IR}$, z-axis) plane, and X is the angle between $\mathbf{k}_P$ and $\mathbf{k}_{IR}$.

The various calculations are iterated until a predetermined accuracy is reached. The line numbers for the appropriate accuracy determining points in the program are listed in Table 1.

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3. Miller, R.C., and Nordland, W.A., IEEE J. Quant. Electr. QE-3:642 (1967)

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Appendix A

PROGRAM LISTING

```
      PROGRAM PHASMCH
      DATA (UP=57.2957795).(DN=.017453293).(CC=6.28318E8).(C1=1000.)
      COMMON/1/W1.W2.W3                        /7/IN3/8/IN2/9/IN1/4/IX/10/PNC
     1/11/K.TCL.TNCL.HWC1.HWC2.HWNC1.HWNC2.HWC3.HWC4.HWNC3.HWNC4.D.J.DIV
     2/12/EXHW1.EXHW2.EXW1.EXW2.EXHW5.EXHW6.FNL1.FNL2.PC1.PNC1.PC2
     3     /5/HW1.HW2.E/13/AX.BX.CX.DX.AY.BY.CY.DY.AZ.BZ.CZ.DZ/2/K1
     4  /6/NX1.NX2.NX3.NY1.NY2.NY3.NZ1.NZ2.NZ3
      REAL LL.L.K1
      REAL NX1.NX2.NX3.NY1.NY2.NY3.NZ1.NZ2.NZ3
      READ 99.N
      DO 9 II=1.N
   50 READ 110.ANAME.UL.LL.L.DL.DU.DD.AX.BX.CX.DX.AY.BY.CY.DY.AZ.BZ.CZ.
     1     DZ
      DELK=6.28318/L
      E=DELK/50.
      DELKK=2.5E-5/L
      IZ1=(DU-DL)/DD+1.
      DO 15 IZZ=1.IZ1
      DDD=DL+DD*(IZZ-1)
      PRINT 100.ANAME.UL.LL.L
      PRINT 102.DDD
      PRINT 101.E.DELK.DELKK
      DO 1 I=1.2
      W1=6943.
      IF(I.EQ.2) W1=10600.
      NX1=FIN(W1.1)
      NY1=FIN(W1.2)
      NZ1=FIN(W1.3)
      W2=LL
      GO TO 11
   10 W2=W2-W2/8.
      IF(W2.LT.UL) GO TO 1
   11 W3=1./(1./W1+1./W2)
      IF(W1.GT.LL.OR.W1.LT.UL.OR.W2.GT.LL.OR.W2.LT.UL.OR.W3.GT.LL.OR.W3
     1      .LT.UL) GO TO 10
      NX2=FIN(W2.1)
      NY2=FIN(W2.2)
      NZ2=FIN(W2.3)
      NX3=FIN(W3.1)
      NY3=FIN(W3.2)
      NZ3=FIN(W3.3)
      DIV=DDD/((NX1+NZ1)/2.)
      D=ASIN((W2/W3)*SIN(DIV +ASIN((W3/W2)*SIN(-DIV ))))*C1
      DO 4 K=1.6
      IF(K.GT.3) GO TO 6
    3 IN3=2
      IN1=1
      IN2=1
      IF(K.EQ.2) IN1=2
      IF(K.EQ.3) IN2=2
      GO TO 12
    6 IN3=1
      IN1=2
      IN2=2
      IF(K.EQ.5) IN1=1
      IF(K.EQ.6) IN2=1
   12 J=1
      TCL=FNX(0..0..0.)
      IF(IX.GT.1) GO TO 5
```

```
      CALL HWDK(L,TCL,0.,4)
      IF(IX.GT.1) GO TO 5
      HWC1=HW1*C1
      HWC2=HW2*C1
      FN=FN2(TCL)
      EXHW1=ASIN(SIN(HW1)*FN)*C1
      EXHW2=ASIN(SIN(HW2)*FN)*C1
      PC1=PVAR(TCL,0.,DIV/2.,0.)
      PC2=PVAR(TCL,0.,-DIV/2.,0.)
      CALL HWDK(L,TCL+DIV/2.,PC1,6)
      HWC3=HW2
      CALL HWDK(L,TCL-DIV/2.,PC2,5)
      HWC4=HW1
      A1=DIV/2.+ABS(PC1+HWC3)
      A2=DIV/2.+ABS(PC2+HWC4)
      FN=FN2(TCL)
      EXW1=ABS(ASIN(SIN( A1 )*FN)+ASIN(SIN( A2 )*FN))*C1
      HWC3=HWC3*C1
      HWC4=HWC4*C1
      FNL1=(A1+A2)*C1/D
      DQ=0.
    7 TNCL=TNCPM(TCL,DQ)
      CALL HWDK(L,TNCL,PNC,3)
      HWNC1=HW1*C1
      HWNC2=HW2*C1
      FN=FN2(TNCL+PNC)
      EXHW5=ASIN(SIN(HW1)*FN)*C1
      EXHW6=ASIN(SIN(HW2)*FN)*C1
      IF(J.EQ.2) GO TO 8
      CALL OUT
      J=2
      DQ=DELK
      TCL=FNX(0.,0.,DQ)
      GO TO 7
    8 PNC1=PVAR(TNCL,PNC,DIV,DELK)
      CALL HWDK(L,TNCL+DIV,PNC1,3)
      HWNC3=HW1
      HWNC4=HW2
      FN=FN2(TNCL+PNC)
      EXW2=ABS(ASIN(SIN(HWNC3)*FN)-ASIN(SIN(HWNC4)*FN))*C1
      HWNC3=HWNC3*C1
      HWNC4=HWNC4*C1
      FNL2=ABS(HWNC3-HWNC4)/D
    5 CALL OUT
    4 CONTINUE
    2 GO TO 10
    1 CONTINUE
   15 CONTINUE
    9 CONTINUE
   99 FORMAT(I2)
  100 FORMAT(20X,21H*** UP-CONVERSION IN ,A5,4H ***//39H THE SHORTEST WA
     1VELENGTH TRANSMITTED IS ,F10.2/38H THE LONGEST WAVELENGTH TRANSMIT
     2TED IS ,F10.2/25H THE LENGTH OF CRYSTAL IS ,F10.2,2X,3HCM./ 90H AL
     3L PHASE MATCH ANGLES ARE IN DEGREES. P.M. ANGLES ARE MEASURED BETW
     4EEN THE Z-AXIS AND K1 /33H TCL = COLINEAR PHASE MATCH ANGLE /37H T
     5NCL = NONCRITICAL PHASE MATCH ANGLE /49H PNC = ANGLE BETWEEN (K1,K
     62) FOR NONCRITICAL P.M. /36H ALL HALFWIDTHS ARE IN MILLIRADIANS / 
     777H HWC = HALFWIDTH OF DELK VS. THETA CURVE MEASURED BETWEEN MAX. 
     8AND FIRST ZERO/39H HWNC = HALFWIDTH FOR  NONCRITICAL P.M. )
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```
102 FORMAT( 68H PC1.PC2 = PC1 FOR TCL = TCL+OR-DIV/2. THE INTERNAL DIV
   1ERGENCE OF W1 / 31H PNC1 = PNC FOR TNCL = TNCL+DIV / 32H EW = EXTE
   2RNAL ANGULAR APERATURE / 32H R = NUMBER OF RESOLVEABLE LINES  /
   349H EXTERNAL DIVERGENCE (FULL ANGLE) OF PUMP BEAM = .F6.4)
101 FORMAT(46H HWC AND HWNC ARE MEASURED WITH TCL/TNCL FIXED /19H DELT
   1A K IS WITHIN .F7.3.15H 1/CM. OF 2PI/L/9H 2PI/L = .F7.3.5X.24H (2P
   2I/L)/K3 IS APPROX = .E10.3/46H K1.K2. AND K3 ARE COPLANER IN THE (
   3Y.Z) PLANE /37H E POLARIZATION IS IN THE (Y.Z) PLANE /41H O POLARI
   4ZATION IS OUT OF THE (Y.Z) PLANE /69H FOR UNIAXIAL CRYSTALS THE Z-
   5AXIS = C-AXIS. NZ = NE. AND NX = NY = NO /26H * INDICATES DELTA K 
   6= 2PI/L /56H W1.W2.W3 CORRESPOND TO PUMP. IR. AND SIGNAL WAVELENG
   7HS /55H NX.NY.NZ ARE REFRACTIVE INDICES FOR X.Y.Z OPTICAL AXIS //5
   89H ALL POSSIBLE UP-CONVERSION PROCESSES ARE LISTED BELOW ----////)
110 FORMAT(A5.5X.6F10.1/4F10.4/4F10.4/4F10.4)
    END
    FUNCTION TNCPM(TCL.D)
    COMMON /10/P
    T=TCL
    DP=0.05
  5 T1=FNX(DP.3.14159.D)
     IF(T1.LT.T) GO TO 10
    DP=-DP
    T1=FNX(DP.3.14159.D)
    IF(T1.LT.T) GO TO 10
    DP=DP/10.
    IF(ABS(DP).LT.0.00001) GO TO 21
    GO TO 5
 10 T=T1
    P=DP
 14 T1=FNX(P+DP.3.14159.D)
    IF(T1.LT.T) GO TO 12
    P=P+DP
    DP=-DP/4.
    IF(ABS(DP).LT.0.0001) GO TO 20
    T=T1
    GO TO 14
 12 P=P+DP
    T=T1
    GO TO 14
 20 TNCPM=T1
    RETURN
 21 TNCPM=T1
    P=DP
    RETURN
    END
    FUNCTION PVAR(T.P.DD.E)
    COMMON/1/W1.W2.W3/2/K1
    REAL K1
    D=.001
    IF(DD.GT.0.0) D=-D
    PP=P
    T1=T+DD
    K1=6.28318E8*FN1(T1)/W1
  1 PP=PP+D
  2 X1=ABS(FDK(T1.PP)+E)
    X2=ABS(FDK(T1.PP+D)+E)
    IF(X2.LT.X1) GO TO 1
    PP=PP+D
    D=-D/5.0
```

```
      IF(ABS(D).LT.1E-5) GO TO 10
      GO TO 2
   10 PVAR=PP
      RETURN
      END
      FUNCTION FNX(XX,PP,DD)
      DATA (E=1.E-6)
      COMMON/1/W1,W2,W3                    /4/IJX
      FK(Q,A)=SQRT((FN1(A)/W1)**2+FN2C(Q)/(W2*W2)-2.*FN1(A)*
     1      SQRT(FN2C(Q))*CXX/(W1*W2))
      D=DD/6.28318E8
      P=PP
      X=XX
      IF(XX.GT.0) GO TO 20
      X=-X
      P=3.14159274+PP
   20 IKX=IJX=0
      D1=ABS(XP-XX)
      XP=XX
      CX=COS(X)
      SX=SIN(X)
      CXX=COS(3.1415927-X)
      CP=COS(P)
      IF(R.LT.0.07.OR.R.GE.1.57) R=0.07
      IF(D1.LT.100.*E) D1=100.*E
      IF(D1.GT.0.1) D1=0.1
      IF(R.EQ.0.07) D1=0.1
    1 IKX=IKX+1
      IF(IKX.GT.100) GO TO 5
      RD=R+D1
      CR=COS(R)
      SR=SIN(R)
      CRD=COS(RD)
      SRD=SIN(RD)
      C2=CR*CX+SR*SX*CP
      C5=CRD*CX+SRD*SX*CP
      Y1=FK(C2,R)
      Y2=FK(C5,RD)
      C3=(FN1(R)/W1-CXX*SQRT(FN2C(C2))/W2)/Y1
      C4=(FN1(RD)/W1-CXX*SQRT(FN2C(C5))/W2)/Y2
      C6=C3*CR+SR*SQRT(ABS(1.-C3*C3))*CP
      C7=C4*CRD+SRD*SQRT(ABS(1.-C4*C4))*CP
      Y=ABS(SQRT(FN3C(C6))/W3+D-Y1)
      Z=ABS(SQRT(FN3C(C7))/W3+D-Y2)
      IF (ABS(D1).LT.  E  )  GO TO 2
      IF(IKX.EQ.1) GO TO 10
      IF (Y.GT.Z)  GO TO 3
      D1=-D1/10
      R=R-10*D1
      GO TO 1
    3 R=R+D1
      IF(R.LT.0.0.OR.R.GE.1.6 ) GO TO 12
      GO TO 1
    2 FNX=R
      IF(C6.LT.0.) IJX=2
      RETURN
   10 IF(Z.GT.Y) D1=-D1
      IF(Z.GT.Y) GO TO 1
      GO TO 3
```

```
 12 IF(IJX.GT.0) GO TO 6
 14 IJX=1
    R=0.07
    D1=0.1
    IKX=1
    GO TO 1
  6 IJX=2
    R=0.0
    RETURN
  5 IF(IJX.GT.0) GO TO 8
    GO TO 14
  8 PRINT 144,X,P,R
    IJX=2
    FNX=R
144 FORMAT(10H FNX ERROR ,5X,3HX = ,F9.4,5X,3HP = ,F9.4,5X,3HR =,F9.4)
    RETURN
    END
    FUNCTION FDK(T,P)
    COMMON /1/W1,W2,W3/2/K1
    REAL K2,K2S,K2C,K3,K1
    DATA (C1=6.28318E8)
    K2=C1*FN2(T+P)/W2
    K2S=K2*SIN(P)
    K2C=K2*COS(P)
    P3=ATAN(K2S/(K1+K2C))
    K3=C1*FN3(T+P3)/W3
    FDK=K3-SQRT(K2S**2+(K2C+K1)**2)
    RETURN
    END
    SUBROUTINE HWDK(L,T,P,I)
    REAL L,L2,K1
    DATA (CC=6.28318E8)
    COMMON /1/W1,W2,W3                  /2/K1/5/V1,V2,E/4/IX
    K1=CC*FN1(T)/W1
    L2=  6.28318/L
    IF(ABS(FDK(T,P)).GT.L2*1.5 ) GO TO 30
    DV=0.01
    IF(I.EQ.2.OR.I.EQ.6) DV=-.01
  1 V=DV
  2 DK=FDK(T,P+V)
    IF(I.GT.3) DK=ABS(DK)
    IF(ABS(     DK -L2).LT.E) GO TO 20
    IF(     DK .LT.L2) GO TO 10
    DV=DV/10.
    IF(ABS(DV).LT.1E-6) GO TO 31
    V=V-DV*9.
    GO TO 2
 10 V=V+DV
    GO TO 2
 20 IF(DV.LT.0.) GO TO 25
    DV=-0.01
    V1=V
    IF(I.EQ.1.OR.I.EQ.5) GO TO 25
    GO TO 1
 25 V2=V
    RETURN
 30 IX=2
 31 PRINT 100,T,P,L2
100 FORMAT(3F15.5)
```

```
      RETURN
      END
      SUBROUTINE OUT
      COMMON/1/W1,W2,W3                    /4/IX/10/PN
     1/11/K,TC,TNC,HW1,HW2,HWN1,HWN2,HW3,HW4,HWN3,HWN4,DD,J,DV
     2/12/EHW1,EHW2,EXW1,EXW2,EHW5,EHW6,FNL1,FNL2,PCL1,PNCL1,PCL2
     3  /6/NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
      REAL NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
      DATA (UP=57.2957795),(DN=.017453293)
      IF(K.GT.1.OR.J.GT.1) GO TO 5
      DIV=DV*1000.
    1 PRINT 100,W1,W2,W3,NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3,DD,DIV
    5 IF(IX.GT.1) GO TO 10
      TCL=TC*UP
      PC1=PCL1*UP
      PC2=PCL2*UP
      TNCL=TNC*UP
      PNC=PN*UP
      PNC1=PNCL1*UP
      IF(J.EQ.2) GO TO 3
      GO TO (11,12,13,14,15,16),K
   11 PRINT 110,TCL,HW1,HW2,EHW1,EHW2,TNCL,PNC,HWN1,HWN2,EHW5,EHW6
      RETURN
   12 PRINT 120,TCL,HW1,HW2,EHW1,EHW2,TNCL,PNC,HWN1,HWN2,EHW5,EHW6
      RETURN
   13 PRINT 130,TCL,HW1,HW2,EHW1,EHW2,TNCL,PNC,HWN1,HWN2,EHW5,EHW6
      RETURN
   14 PRINT 140,TCL,HW1,HW2,EHW1,EHW2,TNCL,PNC,HWN1,HWN2,EHW5,EHW6
      RETURN
   15 PRINT 150,TCL,HW1,HW2,EHW1,EHW2,TNCL,PNC,HWN1,HWN2,EHW5,EHW6
      RETURN
   16 PRINT 160,TCL,HW1,HW2,EHW1,EHW2,TNCL,PNC,HWN1,HWN2,EHW5,EHW6
      RETURN
   10 GO TO (21,22,23,24,25,26),K
   21 PRINT 210
      RETURN
   22 PRINT 220
      RETURN
   23 PRINT 230
      RETURN
   24 PRINT 240
      RETURN
   25 PRINT 250
      RETURN
   26 PRINT 260
      RETURN
    3 PRINT 170,                        TNCL,PNC,HWN1,HWN2,EHW5,EHW6
      PRINT 200,PC1,PC2,HW3,HW4,EXW1,FNL1,PNC1,HWN3,HWN4,EXW2,FNL2
      IF(K.EQ.6) GO TO 4
      RETURN
    4 PRINT 300
      RETURN
  100 FORMAT(6H W1 = ,F10.2,10X,5HW2 = ,F10.2,10X,5HW3 = ,F10.2,10X/3(6H
     1 NX = ,F10.4,9X)/3(6H NY = ,F10.4,9X)/3(6H NZ = ,F10.4,9X)//
     25X, 35HINTERNAL RESOLUTION LIMIT (MRAD) = ,F7.4,
     3 5X, 16HINTERNAL DIV. = ,F6.4//)
  110 FORMAT(9H O+O=E---,7H TCL = , F7.4,2X,6HHWC = ,4F6.1,2X,7HTNCL = ,
     1       F7.4,2X,6HPNC = , F7.4,2X,7HHWNC = ,4F6.1)
  120 FORMAT(9H E+O=E---,7H TCL = , F7.4,2X,6HHWC = ,4F6.1,2X,7HTNCL = ,
```

```
  1       F7.4,2X,6HPNC =   F7.4,2X,7HHWNC = ,4F6.1)
130 FORMAT(9H O+E=E---,7H TCL = , F7.4,2X,6HHWC = ,4F6.1,2X,7HTNCL = ,
  1       F7.4,2X,6HPNC =   F7.4,2X,7HHWNC = ,4F6.1)
140 FORMAT(9H E+E=O---,7H TCL = , F7.4,2X,6HHWC = ,4F6.1,2X,7HTNCL = ,
  1       F7.4,2X,6HPNC = , F7.4,2X,7HHWNC = ,4F6.1)
150 FORMAT(9H O+E=O---,7H TCL = , F7.4,2X,6HHWC = ,4F6.1,2X,7HTNCL = ,
  1       F7.4,2X,6HPNC = , F7.4,2X,7HHWNC = ,4F6.1)
160 FORMAT(9H E+O=O---,7H TCL = , F7.4,2X,6HHWC = ,4F6.1,2X,7HTNCL = ,
  1       F7.4,2X,6HPNC = , F7.4,3X,7HHWNC = ,4F6.1)
170 FORMAT(9H    *    ,  4F6.                              7HTNCL = ,
  1       F7.4,2X,6HPNC = , F7.4,2X,7HHWNC = ,4F6.1)
200 FORMAT(10X,5HPC1 =,F7.4,6H PC2 =,F7.4,5H HW =,2F6.1,4H EW=,F6.1,
  1    3H R=,F5.1,7H PNC1 =,F7.4,5H HW =,2F6.1,4H EW=,F6.1,3H R=,F6.1)
210 FORMAT(9H O+O=E---,10X,28H*** NOT PHASE MATCHABLE ***   )
220 FORMAT(9H E+O=E---,10X,28H*** NOT PHASE MATCHABLE ***   )
230 FORMAT(9H O+E=E---,10X,28H*** NOT PHASE MATCHABLE ***   )
240 FORMAT(9H E+E=O---,10X,28H*** NOT PHASE MATCHABLE ***   )
250 FORMAT(9H O+E=O---,10X,28H*** NOT PHASE MATCHABLE ***   )
260 FORMAT(9H E+O=O---,10X,28H*** NOT PHASE MATCHABLE ***  //39(3H  *)
  1       //)
300 FORMAT(//30(3H  *)//)
    END
    FUNCTION FIN(W,I)
    COMMON/13/AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ
    X=W*W*1.0E-8
    GO TO (1,2,3),I
  1 FIN=SQRT(AX+BX/(X-CX)-DX*X)
    RETURN
  2 FIN=SQRT(AY+BY/(X-CY)-DY*X)
    RETURN
  3 FIN=SQRT(AZ+BZ/(X-CZ)-DZ*X)
    RETURN
    END
    FUNCTION FN1(A)
    COMMON /1/W1,W2,W3                       /9/IN1
   4  /6/NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
    REAL NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
    GO TO (1,2),IN1
  1 FN1=NX1
    RETURN
  2 FN1=1./SQRT((COS(A)/NY1)**2+(SIN(A)/NZ1)**2)
    RETURN
    END
    FUNCTION FN2(A)
    COMMON /1/W1,W2,W3/8/IN2
   4  /6/NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
    REAL NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
    GO TO (1,2),IN2
  1 FN2=NX2
    RETURN
  2 FN2=1./SQRT((COS(A)/NY2)**2+(SIN(A)/NZ2)**2)
    RETURN
    END
    FUNCTION FN3(A)
    COMMON /1/W1,W2,W3/7/IN3
   4  /6/NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
    REAL NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
    GO TO (1,2),IN3
  1 FN3=NX3
```

```
      RETURN
    2 FN3=1./SQRT((COS(A)/NY3)**2+(SIN(A)/NZ3)**2)
      RETURN
      END
      FUNCTION FN2C(Q)
      COMMON /1/W1,W2,W3/8/IN2
     4  /6/NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
      REAL NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
      GO TO (1,2),IN2
    1 FN2C=NX2*NX2
      RETURN
    2 FN2C=1./((Q/NY2)**2+(1.-Q*Q)/NZ2**2)
      RETURN
      END
      FUNCTION  FN3C(Q)
      COMMON /1/W1,W2,W3/7/IN3
     4  /6/NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
      REAL NX1,NX2,NX3,NY1,NY2,NY3,NZ1,NZ2,NZ3
      GO TO (1,2),IN3
    1 FN3C=NX3*NX3
      RETURN
    2 FN3C=1./((Q/NY3)**2+(1.-Q*Q)/NZ3**2)
      RETURN
      END
```

Appendix B

SAMPLE INPUT DATA AND PROGRAM OUTPUT

```
  1
HGS        6300.      135000.    1.0          .001       .001       .001
6.9445     .36579     .135665    .00192511
6.9445     .36579     .135665    .00192511
8.39218    .538991    .138793    .00266768


          *** UP-CONVERSION IN HGS   ***

THE SHORTEST WAVELENGTH TRANSMITTED IS   6300.00
THE LONGEST WAVELENGTH TRANSMITTED IS 135000.00
THE LENGTH OF CRYSTAL IS       1.00  CM.
ALL PHASE MATCH ANGLES ARE IN DEGREES, P.M. ANGLES ARE MEASURED BETWEEN THE Z-AXIS AND K1
TCL =COLLINEAR PHASE MATCH ANGLE
TNCL = NONCRITICAL PHASE MATCH ANGLE
PNC = ANGLE BETWEEN (K1,K2) FOR NONCRITICAL P.M.
ALL HALFWIDTHS ARE IN MILLIRADIANS
HWC = HALFWIDTH OF DELK VS. THETA CURVE MEASURED BETWEEN MAX. AND FIRST ZERO
HWNC = HALFWIDTH FOR NONCRITICAL P.M.
PC1,PC2 = PC1 FOR TCL = TCL+EW-DIV/2, THE INTERNAL DIVERGENCE OF W1
PNC1 = PNC FOR TNCL = TNCL+DIV
EW = EXTERNAL ANGULAR APERTURE
R = NUMBER OF RESOLVABLE  LINES
EXTERNAL DIVERGENCE (FULL ANGLE) OF PUMP BEAM = 0.0010
HWC AND HWNC ARE MEASURED WITH TCL/TNCL FIXED
DELTA K IS WITHIN   0.126 1/CM. OF 2PI/L
2PI/L =    6.283       (2PI/L)/K3 IS APPROX =  2.500-005
K1,K2, AND K3 ARE COPLANAR IN THE (Y,Z) PLANE
E POLARIZATION IS IN THE (Y,Z) PLANE
O POLARIZATION IS OUT OF THE (Y,Z) PLANE
FOR UNIAXIAL CRYSTALS THE Z-AXIS = C-AXIS, NZ = NE, AND NX = NY = NO
* INDICATES DELTA K = 2PI/L
W1,W2,W3 CORRESPOND TO PUMP, IR, AND SIGNAL WAVELENGTHS
NX,NY,NZ ARE REFRACTIVE INDICES FOR X,Y,Z OPTICAL AXIS

ALL POSSIBLE UP-CONVERSION PROCESSES ARE LISTED BELOW ----
```

```
W1 =    6943.00          W2 =  135000.00          W3 =    6603.39
NX =     2.8284          NX =     2.5682          NX =     2.8568
NY =     2.8284          NY =     2.5682          NY =     2.8568
NZ =     3.1561          NZ =     2.8123          NZ =     3.1944

    INTERNAL RESOLUTION LIMIT (MRAD) =  6.4982     INTERNAL DIV. = 0.3342

O+O=E---          *** NOT PHASE MATCHABLE ***
E+O=E---          *** NOT PHASE MATCHABLE ***
O+E=E---          *** NOT PHASE MATCHABLE ***
E+E=O--- TCL  = 22.4966  HWC =    9.0  -8.3  23.4 -21.6  TNCL = 22.4185  PNC =  3.9503  HWNC =   38.0 -32.0  99.4 -83.6
   *                                                     TNCL = 22.4371  PNC =  3.9503  HWNC =   52.0 -47.0 136.2-143.0
        PC1 =-0.2613 PC2 = 0.2521 HW =  -7.6  10.0 EW=  69.9 R=  4.1 PNC1 = 2.0710 HW =  96.2 -25.3 EW= 319.6 R=  18.7
O+E=O---          *** NOT PHASE MATCHABLE ***
E+O=O--- TCL  = 22.9481  HWC =   33.3 -33.3  85.6 -85.6  TNCL = 22.9477  PNC = -0.1970  HWNC =   37.0 -30.0  95.1 -77.1
   *                                                     TNCL = 22.9666  PNC = -0.0501  HWNC =   47.6 -46.0 122.5-118.4
        PC1 =-1.3820 PC2 = 0.0023 HW = -17.0  24.0 EW= 168.4 R= 10.1 PNC1 =-1.9086 HW =  91.0 -24.1 EW= 297.5 R=  17.7

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NCI

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A COMPUTER PROGRAM TO ANALYZE OPTICAL PARAMETRIC UP-CONVERSION PROCESSES IN NONLINEAR CRYSTALS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

This is an interim report; work on the problem is continuing.

5. AUTHOR(S) (First name, middle initial, last name)

R.A. Andrews

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
October 27, 1969	23	5

8a. CONTRACT OR GRANT NO. NRL Problem N01-14	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 6941
b. PROJECT NO. XF-52545002-8063	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Department of the Navy (Naval Electronic Systems Command), Washington, D.C. 20360

13. ABSTRACT

A Fortran computer program has been developed which analyzes a nonlinear material to determine (a) the range of IR wavelengths that can be converted in a phase matched (PM) process, (b) the PM orientation of the wave vectors for critical and noncritical PM, (c) the angular aperture for PM conversion, and (d) the maximum number of resolvable lines for image conversion. These characteristics are determined as a function of IR wavelength for a given pump wavelength, pump radiation divergence, and length of nonlinear crystal.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Optical parametric processes Up-Conversion Nonlinear crystals Quantum optics						